A Laguerre-based Distributed Nonlinear Model Predictive Control Scheme for Dynamic Obstacle Avoidance under Denial of Service Attacks

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Nonlinear Model Predictive Control

Key Part: Uses a Model of the System to Predict and Optimise the Future Performance

Some Applications:

- Path Planning/Obstacle Avoidance
- Optimising Energy Usage/Generate
- Distributed/Decentralised Control
- Robust/Stochastic Control (Handling Uncertainty)
- Adaptive and Fault Tolerant Control

Advantages:

- Ability to Anticipate Future Events
- Non-reactive control -> Smoother Control Actions
- Handling Time-Delays, Nonlinear Dynamics and Constraints

Disadvantages/Challenges:

- Requires a relatively accurate model of the system
- Computational burden -> Impacts real-time capabilities

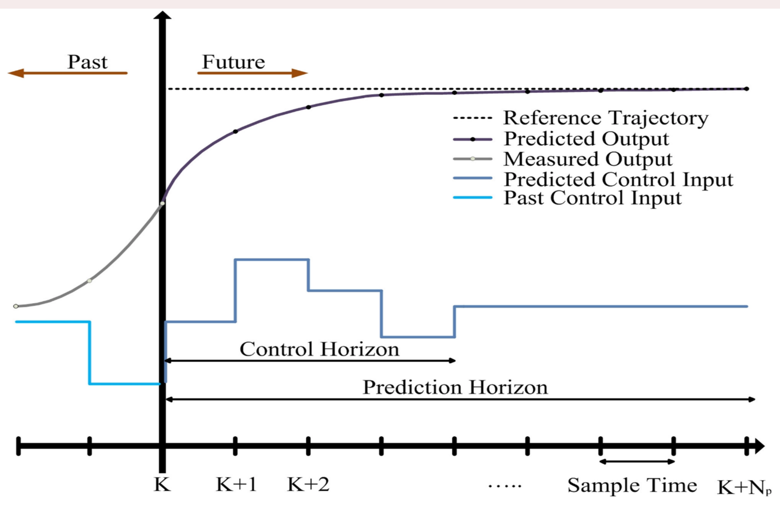


Figure 1: Predictive framework taken from [1]

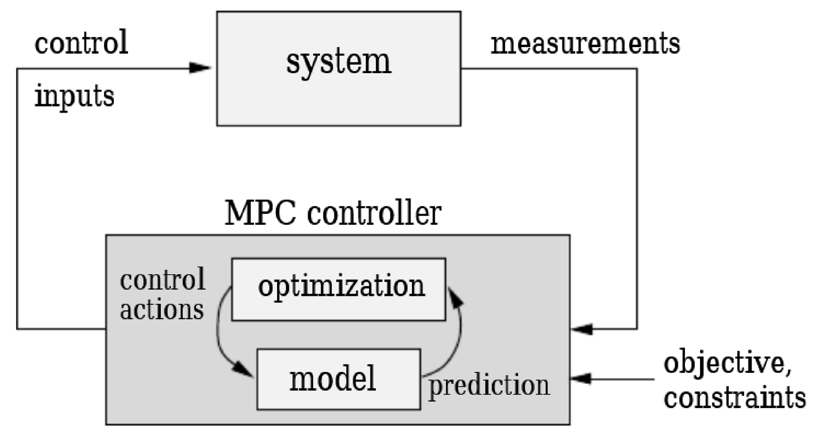


Figure 2: Predictive Control Loop taken from [2]

Proposed DNS Potential Fields

Two Common Methods:

- Potential Fields (Current)
- Nonlinear Constraints

$$\hat{P}_k^i = \frac{k_{pot}(DNS)}{(\hat{d}_k)_i - d_{min}(DNS)} \quad \text{vs} \quad (\hat{d}_k)_i \geq d_{min}(DNS)$$

Advantages of DNS Potential Field:

- Adaptive Obstacle Avoidance parameters according to communication performance metrics.

Disadvantages:

- Sub-optimal w.r.t. NL Constraints

<u>**Predicted Distance to obstacles**</u>

$$(\hat{d}_k) = \sqrt{\left(\hat{x}_k - \hat{x}_{obs_k^i}\right)^2 + \left(\hat{y}_k - \hat{y}_{obs_k^i}\right)^2 + \left(\hat{z}_k - \hat{z}_{obs_k^i}\right)^2}$$

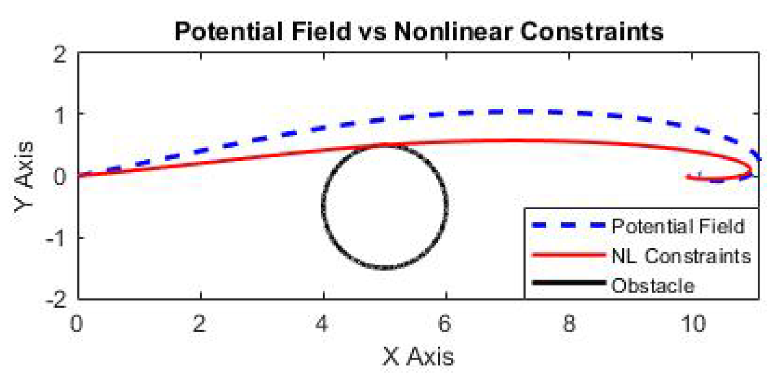


Figure 3: Comparison of Potential Field vs Nonlinear Constraints

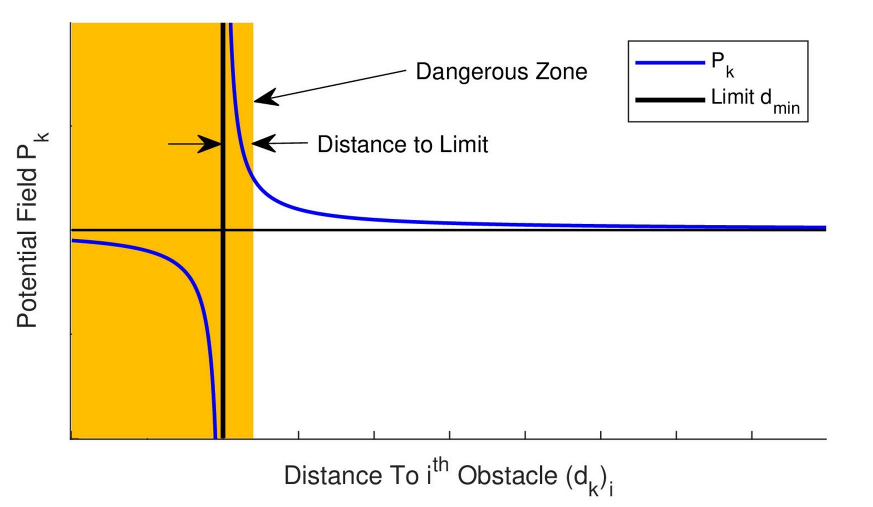


Figure 4: Potential Field graph depicting a "dangerous-zone" in yellow.

Distributed Model Predictive Control

Advantages vs Centralised:

- Reduced computational burden
- Resilient to local failures, inc. communications
- Scalability
- Modularity
- Reconfigurability

Disadvantages vs Centralised:

- Suboptimal Solution
- Relies on communication

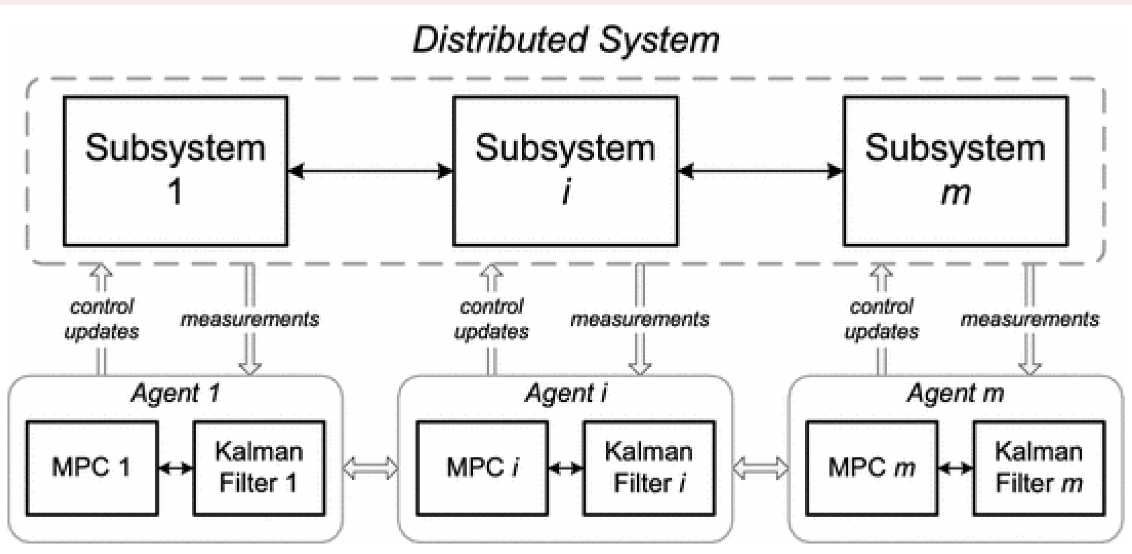


Figure 4: Distributed Predictive Control framework taken from [3]

Laguerre Polynomials as Parametric Curves

Key Part: Rather than using single waypoints, use a "compressed" smooth representation of the planned trajectory.

Advantages:

- Reduced computational burden
- Smooth trajectories with competitive performance if appropriately tuned
- Capture a large plan with few variables
- Less bandwidth requirements
- Less information to encrypt/decrypt

Disadvantages:

- Can be slightly suboptimal
- Less flexible due to limited options

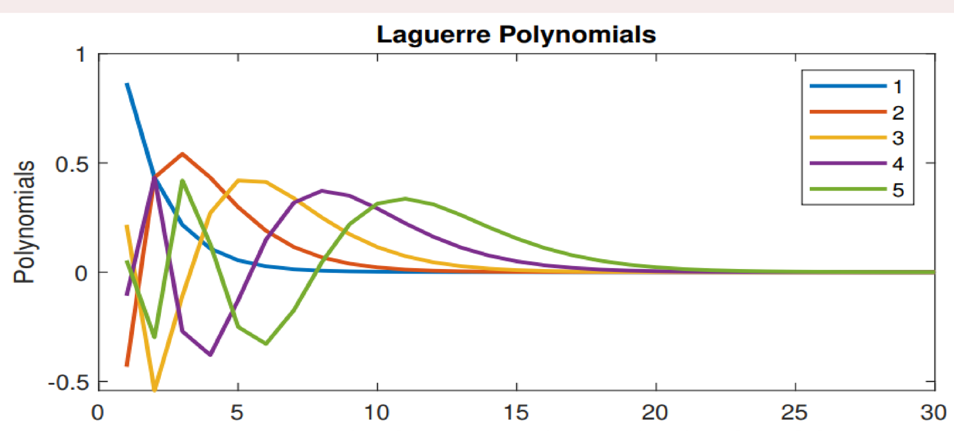


Figure 5: Example of Laguerre Polynomials taken from [4]

<u>**Laguerre Dynamics**</u>

$$\begin{bmatrix} L(1) \\ L(2) \\ \vdots \\ L(N_L) \end{bmatrix}_{k+1} = \begin{bmatrix} a_L & 0 & \cdots & 0 \\ \beta & a_L & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ (-a_L)^{N_L-2}\beta & \ddots & \ddots & a_L \end{bmatrix} \begin{bmatrix} L(1) \\ L(2) \\ \vdots \\ L(N_L) \end{bmatrix}_k$$

Parameterised NMPC for Obstacle Avoidance under Denial of Service Attacks

<u>**Input Parameterised Cost Function**</u>

$$J = (X_r - \hat{X})^T Q (X_r - \hat{X}) + (U_r - \hat{U})^T R (U_r - \hat{U})$$
$$\begin{aligned} st. \quad & \hat{x}_k = x_0 \\ & \hat{x}_{k+i} = f(\hat{x}_{k+i-1}, \hat{u}_{k+i-1}) \quad \forall i = [1, ..., N_p] \\ & \hat{U} = \mathbb{N}\hat{\eta} \\ & X_{min} \leq \hat{X} \leq X_{max} \\ & U_{min} \leq \hat{U} \leq U_{max} \end{aligned}$$

<u>**Input Parameterised Quadratic Program**</u>

$$J = \hat{\eta}^T E \hat{\eta} + 2 f^T \hat{\eta} \quad st. \quad M\hat{\eta} \leq \gamma$$
$$E = \mathbb{N}^T (H^T Q H + R) \mathbb{N}$$
$$f = -\mathbb{N}^T \left[H^T Q (X_r - \bar{X} + H\bar{U}) - R(\bar{U} - U_r) \right]$$
$$M = \begin{bmatrix} H\mathbb{N} \\ -H\mathbb{N} \\ \mathbb{N} \\ -\mathbb{N} \end{bmatrix} \quad \gamma = \begin{bmatrix} X_{max} - \bar{X} + H\bar{U} \\ -(X_{min} - \bar{X} + H\bar{U}) \\ U_{max} \\ -U_{min} \end{bmatrix}$$

<u>**Modified Cost Function with DNS Potential Fields**</u>

$$J_{total} = J + J_{pot} = J + \min \sum_{i=1}^{N_{obs}} \left(\sum_{k=1}^{N_p} P_k^i \right)$$

<u>**Modified Linear Term with Potential Fields**</u>

$$f_{total} = f + \sum_{i=1}^{N_{obs}} \left[(H_{\mathbb{N}})_i^T \mathbf{1}^{N_p} \right]$$

<u>**Input Parameterised Prediction Models**</u>

$$\hat{U} = \mathbb{N}\eta = \bar{U} + \delta\hat{U}$$
$$\hat{X} = \bar{X} + H\delta\hat{U} = \bar{X} + H(\mathbb{N}\eta - \bar{U})$$
$$H = \begin{bmatrix} h_{1,1} & 0 & \cdots & 0 \\ h_{2,1} & h_{2,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ h_{N_p,1} & h_{N_p,2} & \cdots & h_{N_p,N_p} \end{bmatrix}$$
$$h_{k,j} = \begin{cases} B_{k-1} & k = j \\ A_{k-1} h_{k-1,j} & k > j \end{cases}$$

<u>**Input Parameterisation Matrix**</u>

$$\mathbb{N} = \left[\begin{array}{ccc} (L_0^1)^T & 0 & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & 0 & (L_0^{n_u})^T \\ \hline \vdots & \vdots & \vdots \\ \hline (L_{N_p-1}^1)^T & 0 & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & 0 & (L_{N_p-1}^{n_u})^T \end{array} \right] = \left[\begin{array}{c} \mathbb{N}_0 \\ \hline \vdots \\ \hline \mathbb{N}_{N_p-1} \end{array} \right]$$

<u>**Input->State Parameterised Prediction Matrix**</u>

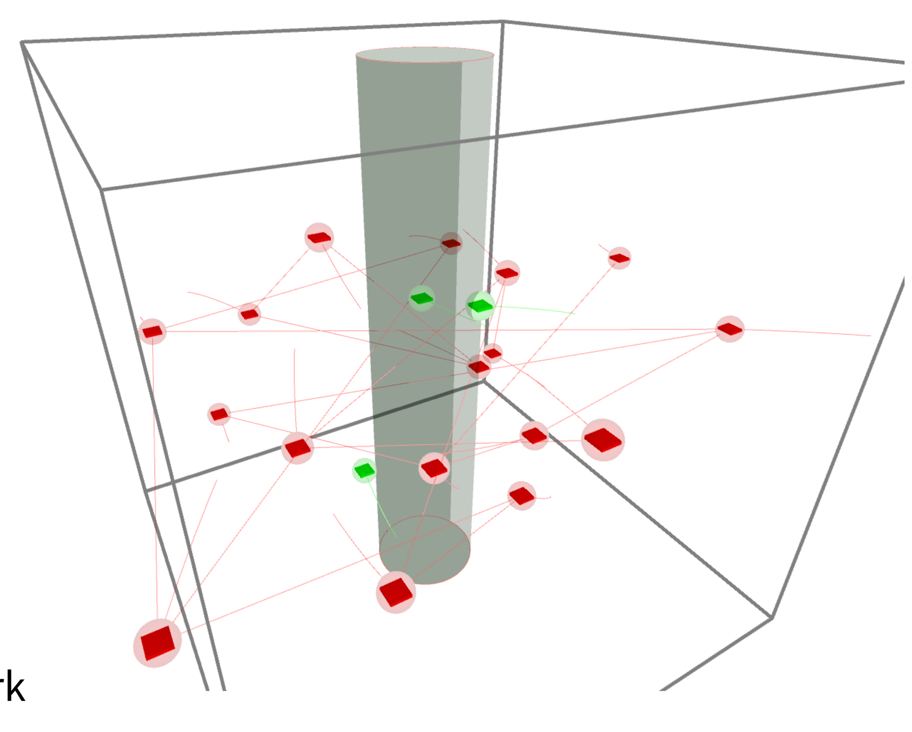
$$H_{\mathbb{N}} = \left[(h_{\mathbb{N}})_1^T \quad (h_{\mathbb{N}})_2^T \quad \cdots \quad (h_{\mathbb{N}})_{N_p}^T \right]^T$$
$$(h_{\mathbb{N}})_k = \begin{cases} B_{k-1}\mathbb{N}_{k-1} & k = 1 \\ A_{k-1}(h_{\mathbb{N}})_{k-1} + B_{k-1}\mathbb{N}_{k-1} & k > 1 \end{cases}$$

Multi-Rotor Case Study: Simulation Results

Case Study: Large-scale obstacle avoidance/ deconfliction of 20 Multi-rotor UAVs within a cylindrical airspace of 15 meters radius with a height of 10 meters and a DNS attack on a designated area.

Simulation Assumptions/Specifications:

- <u>Model:</u> Double-Integrator with acceleration inputs, commonly used for path planning.
- <u>Target:</u> Randomly generated waypoints changing approximately every 3 seconds.
- <u>Path Planning Condition:</u> Maintain a minimum distance of 1 meter to other UAVs.
- <u>Communication:</u> Available through the network when not disrupted by the DNS attacks. UAVs transmit state and parameterised trajectories.



Full Animation: https://youtu.be/GP8TDaHymjg

Figure 8: Large-scale Obstacle Avoidance for UAVs under DNS attack. DNS area visible in grey cylinder. Affected UAVs visible in green.

Future Work

- Develop efficient auto-generated algorithms for its implementation.
- Increase the model accuracy to couple with inner UAV dynamics and control systems, as well as to model noise in the system.
- Develop an indoor and outdoor experimental validation of the proposed approach.
- Increase the level of uncertainty from the environment including common GPS positioning errors such as drift, scales or bias, as well as other positioning errors obtained from other sensors such as cameras, optical flow or LIDAR.
- Extend approach to handle Distributed Denial of Service (DNSS), as well as other cyber-physical attacks such as GPS/Positioning spoofing and jamming, and propose legible[5] MPC solutions for anomaly detection, monitoring and deconfliction between UAVs.

Key Questions

- How to develop efficient, reliable and secure communication between UAVs for obstacle avoidance?
- How to develop legible control systems which would allow analysis and monitoring from an **external observer** for cyber-security purposes within the context of obstacle avoidance?
- What information should autonomous vehicles share that would allow anomaly detection and monitoring of cyber-physical attacks **from an external point of view?**
- How to use legibility and correlation between UAVs control signals to achieve efficient encrypted communication?

References

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[4] O. Gonzalez Villarreal, "Efficient Real-Time Solution for Nonlinear Model Predictive Control with Applications". Ph. D. Thesis, University of Sheffield, August 2021
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